Power Control in Gaussian Frequency- Flat Interference Relay Channels via Game Theory

Shalini Suman

Abstract— We address the power control problem in Gaussian frequency-flat interference relay channels using a game-theoretic framework. Due to the competitive nature of the multi-user environment, we model the problem as a strategic non-cooperative game and show that this game always has a unique Nash Equilibrium (NE), for any system profile. Two iterative algorithms, based on simultaneous and asynchronous algorithm. We also prove that the proposed algorithm always converges to the unique NE from an arbitrary starting point. We consider here *Gaussian interference relay* games (GIRGs), where instead of allocating the power budget across a set of sub-channels, each player aims to decide the optimal power control strategy across a set of hops. We show that the GIRG always possesses a unique NE for a two-player version of the game, irrespective of any channel realization or initial system parameters such as power budgets and noise power. We then conclude that the distributed game-theoretic approach exhibits great potential in the context of interference relay. More importantly, the global optimality in terms of the sum information rate is achieved by the NE, when the interference is relatively low.

Index Terms— Nash Equilibrium (NE), Power control problem, Gaussian interference relay games (GIRGs), Power Control, Game

1 INTRODUCTION

Cooperative communication is a far dated concept originating from the work on relay channel [1]. By endeavor of [2] [3], cooperative communication came back to the limelight about 20 years later. Higher through put & reliability could be obtained in wireless cooperative network as neighboring node serve as intermediate relays after overhearing the signal transmitted from a source, which in turn aid in forwarding the message to the destination, in order to create spatial diversity.

Studies on resource control, especially power allocation for cooperative networks have been conducted under various levels of channel knowledge [4]. The present work scenario concentrates on single-user relay networks. In this system, only a signal transmission link is active at a time-point which is most probably aided by one or more relays. But in the case of multi-user relay network having multiple simultaneous transmissions, interference between the transmission links. This is mainly due to the fact that increase of one link's performance has to be compensated by the degradation of another link's performance. Hence, the focal point of the work is to know how to balance the tradeoff between different links in order to achieve satisfactory network-wise utility.

Game theory has emerged as a very powerful tool because it provides a convenient framework to study interactions among self-interested individuals. Game-theoretic approaches relax the objective function to achieve global

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Current game theory works on power control have evolved from scalar games, where each user only has one degree of freedom (typically the transmit power) for optimization, to more complicated vector games, where each user only has one degree of freedom (typically the transmit power) for optimization, to more complicated vector games, where each user has several degrees of freedom such as user codes or power allocation across a set of sub-channels. A typical problem setting for the scalar game is the CDMA uplink power control, which has been thoroughly investigated by a plethora of works [7], [8], etc. Scalar power control has also been studied from the jamming perspective and modeled as a dynamic jamming game in [9]. A typical problem setting for the vector game is the Gaussian frequency-selective Interference channel, which has been studied in [10], [11]. A vector power control problem in flat-fading Gaussian interference channels. Since these vector games may admit multiple Nash equilibria (NEs), tremendous efforts have been devoted to characterize sufficient conditions for the uniqueness of the NE, with the broadest one reported in [12], which are also the sufficient conditions that guarantee global convergence

In this paper, we address the power control problem in the multi-cell multi-hop cellular systems using a game-theoretic approach. We adopt the same model as [16] and maximize the throughput of a two-cell system, by optimally controlling the power for both the sources and the relays. The main contributions of this paper are twofold. We propose a new type of power control game, which we call the Gaussian interference relay game (GIRG), within the interference relay channel setting. We show that the GIRG always possesses a unique NE, irrespective of the channel realization and the initial system parameters such as power budgets and noise power. To benchmark the performance of NE, we investigate the sum-rate optimization problem, which is non-convex and non-smooth in nature.

The rest of this paper is organized as follows. In Section 2, the system model is described and the power allocation problem is formulated as a game. We then prove the existence and uniqueness of the NE in Section 3. In Section 4, two distributed algorithms are proposed to achieve the unique NE and their convergence properties are proved therein. Numerical examples are presented in Section 5. Finally, we conclude this work in Section 6.

2. SYSTEM MODEL AND PROBLEM FORMULATION:

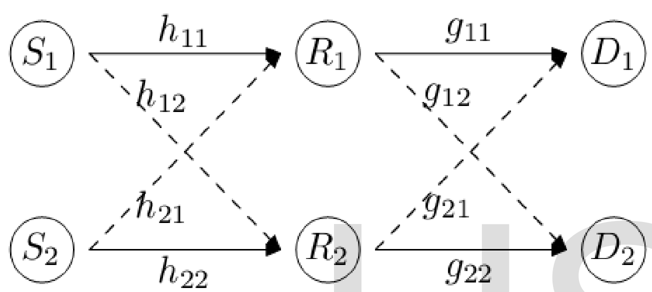

Fig. 1: System model.

2.1 SYSTEM MODEL

We address the problem of power allocation for interference relay channels, as depicted in Fig.1, which consists of two single-relay-assisted communication links. That is, link1 and link2 $S_1 \rightarrow R_1 \rightarrow D_1$ and $S_2 \rightarrow R_2 \rightarrow D_2$ share the same physical resources, such as time and bandwidth. Each source Si tries to communicate, via a repetition-based decode-and-forward (DF) protocol, with its corresponding base station Di, assisted by a dedicated half-duplex relay R_i. In principle, S1 (R1) causes interference to S2 (R2), and vice versa.

At the first time slot, the signal received at R1 is

$$y_{R_1} = h_{11}\sqrt{p_{11}}x_1 + h_{21}\sqrt{p_{21}}x_2 + n_{11}, \qquad (1)$$

Where $\boldsymbol{x_1}$ and $\boldsymbol{x_2}$ are the symbols transmitted by S_1 and S_2, Respectively, and $E|\boldsymbol{x_1}|^2 = E|\boldsymbol{x_2}|^2 = 1$. At the second time slot, each relay forwards the successfully decoded symbol to its destination, so that the received signal at $\mathbf{D_1}$ is

$$y_{D_1} = g_{11}\sqrt{p_{12}}x_1 + g_{21}\sqrt{p_{22}}x_2 + n_{12}. \qquad (2)$$

The channel coefficients between S_i and R_j, and the coefficients between R_i and D_j, are denoted as h_{ij} and g_{ij}, i, j ∈ {1, 2}, respectively, n_{i1} is the additive white Gaussian noise (AWGN) at R_i with variance σ^2_{i1}, and n_{i2} at D_i with variance σ^2_{i2}. P_{i1} and p_{i2} denote the transmit power at S_i and R_i, respectively. The received signals at R2 and D2 have the symmetrical structure as (1) and (2).

The channel is assumed to change sufficiently slowly so that the information theoretical results are meaningful. Besides, we assume that there is a total power constraint on each communication pair ($S_i \rightarrow R_i \rightarrow D_i$), which corresponds to the maximum power that a given packet is allowed to consume throughout its propagation from source to destination. Based on the above assumptions, the maximum achievable rate of link1, given link2 as interference, and that of link 2, given link 1 as interference, are

$$\begin{aligned} I_1 &= \frac{1}{2}\min\left\{\log_2\left(1+\frac{p_{11}}{p_{21}\alpha_{21}+\omega_{11}}\right),\right. \\ &\left.\log_2\left(1+\frac{p_{12}}{p_{22}\beta_{21}+\omega_{12}}\right)\right\}, \qquad (3) \\ I_2 &= \frac{1}{2}\min\left\{\log_2\left(1+\frac{p_{21}}{p_{11}\alpha_{12}+\omega_{21}}\right),\right. \\ &\left.\log_2\left(1+\frac{p_{22}}{p_{12}\beta_{12}+\omega_{22}}\right)\right\}, \qquad (4) \end{aligned}$$

Where $\omega i1 = \sigma^2_{i1} / |h_{ii}|^2$ and $\omega_{i2} = \sigma^2_{i2} / |gii|^2$ denote the normalized background noise power at the relays and the destinations, respectively, for i ∈ Ω.
$\alpha_{ij} = |h_{ij}|^2 / |h_{jj}|^2$ and $\beta_{ij} = |g_{ji}|^2 / |g_{jj}|^2$ denote the normalized interference coefficients at the relays and destinations, respectively, for i = j, i, j ∈ Ω.

2.2 Problem Formulation

The central design task is to determine a transmit power Control policy (p_{i1}, p_{i2}) for each communicating link to achieve a satisfactory network-wide utility. This seemingly simple problem is highly complicated due to the interference coupling between the two links, because generally, increasing the transmit-power level of one link has the undesirable effect of also increasing the levels of interference to the other link. The centralized approach corresponding to the following optimization problem

$$(\mathbf{P1}) \quad \underset{\mathbf{p}_i}{\text{maximize}} \quad I_1 + I_2 \\ \text{subject to} \quad \mathbf{p}_i \in \mathcal{S}_i, \quad \forall i \in \Omega, \qquad (5)$$

Where $P_i = [p_{i1}, p_{i2}]^T$ denotes the strategy adopted by link i, ∀i ∈ Ω. S_i is the admissible strategy set of link i, defined as $S_i = \{p_i : 0 \le p_{i1} + p_{i2} \le P_i\}$, ∀i ∈ Ω,
i.e., a sum power constraint on each link.
Problem (P1) is fundamentally a non-convex problem, Whose global optimal solution cannot be efficiently found,

even in a centralized fashion, needless to say the heavy signaling burden for centralized control. Nevertheless, the inherent competitive nature of (P1) motivates us to use the convenient framework of game theory. Specifically, we model the problem as a pure strategic non-cooperative game, in which the players are links and the payoff functions are their own rates. Each player competes against the other by choosing the power control strategy that maximizes its own rate, subject to a sum power constraint. Mathematically, the game can be expressed as

$$\mathcal{G}: \quad \begin{array}{ll} \underset{\mathbf{p}_i}{\text{maximize}} & I_i(\mathbf{p}_i, \mathbf{p}_{-i}) \\ \text{subject to} & \mathbf{p}_i \in \mathcal{S}_i \end{array} \quad \forall i \in \Omega, \tag{6}$$

Where $p_{-i} = [p_{j1}, p_{j2}]^T$, i= j, i, j ∈ Ω. Note that the solution to (6) is the well-known NE.

Based on the above game-theoretic formulation, we adopt competitive optimality as the optimization criterion. That is, the achievement of a NE, which is reached when each user, given the power control strategy of the other, does not get any rate increase by deviating from the power strategy corresponding to the equilibrium. Formally stated, we have

Definition: A pure strategy profile $p^* = (p^*)_{i\in\Omega} \in S$ is a NE of game G if

$$I_i(\mathbf{p}_i^*, \mathbf{p}_{-i}^*) \geq I_i(\mathbf{p}_i, \mathbf{p}_{-i}^*), \quad \forall \mathbf{p}_i \in \mathcal{S}_i, \ \forall i \in \Omega, \tag{7}$$

where $\mathcal{S} = \mathcal{S}_1 \times \cdots \times \mathcal{S}_{|\Omega|}$, and $|\Omega|$ denotes the cardinality of the set Ω.

3. EXISTENCE AND UNIQUENESS OF THE NASH EQUILIBRIUM

In this section, we show that the NE of game G not only exists but is always unique.

3.1 Existence of the Nash Equilibria

The existence of NE for game G is guaranteed by the following fundamental theorem in game theory [15]:

Theorem 1: A strategic non-cooperative game G = {Ω, $\{S_i\}_{i\in\Omega}$, $\{I_i\}_{i\in\Omega}$ } admits at least one NE if, ∀i ∈ Ω: 1):

• *The set Si is a nonempty compact convex subset of an Euclidean space;*
• *The payoff function Ii* (**p***i*, **p**−*i*) *is continuous on S and quasi concave on Si.*

Based on Theorem 1, we can conclude that game G of (6) always admits at least one NE, since the strategy space of each user is a convex and compact set, and the payoff function of each user is continuous in S and concave in pi ∈ Si, hence quasi-concave.

3.2 Uniqueness of the Nash Equilibrium

While the current literature on Gaussian interference games indicates that the uniqueness of NE is only guaranteed by sufficient conditions, we will show that the GIRG considered here always possesses a unique NE for any system profile.

Let $p = [p_1^T, p_2^T]^T$, and assume that p* is a NE of game G. according to the definition of NE, $p1^*$ is the maximizer of I_1 (**p**1, **p**2*), and **p**2* the maximizer of I_2 (p2, p1*), which are given by the well-known equal rate relation as follows:

$$\frac{p_{11}^*}{p_{21}^*\alpha_{21} + \omega_{11}} = \frac{p_{12}^*}{p_{22}^*\beta_{21} + \omega_{12}}, \tag{8}$$

$$\frac{p_{21}^*}{p_{11}^*\alpha_{12} + \omega_{21}} = \frac{p_{22}^*}{p_{12}^*\beta_{12} + \omega_{22}}, \tag{9}$$

Plus the two sum power constraints

$$p_{11}^* + p_{12}^* = P_1, \tag{10}$$

$$p_{21}^* + p_{22}^* = P_2. \tag{11}$$

Hence, we have four equations characterizing the NE **p***. In other words, the set of NE corresponds to the solution set of Eqns. (8)-(11). By showing that there always exists only one feasible solution, we have the following theorem.

Theorem 2: *Game G always possesses a unique NE.*
Proof: See Appendix A.

4. DISTRIBUTED ASYNCHRONOUS ALGORITHM AND CONVERGENCE

So far, we have shown that game *G* always has a unique NE, irrespective of the channel conditions, network topologies, and power budgets. Since there is no reason to expect a system to be initially at equilibrium, the concept of equilibrium is meaningful in practice only if one is able to find a procedure that reaches such an equilibrium from non-equilibrium states. We propose a distributed asynchronous algorithm that performs this task and prove that it always converges to the NE.

In this section, we first define an iterative mapping, based on which two decentralized asynchronous algorithms are proposed. Next, we study the convergence properties of both algorithms and show that they always converge to the unique NE. Synchronous algorithms share the common requirements of certain forms of synchronization and coordination between different links, since both users need to update their strategies according to a given schedule (i.e., either sequentially or simultaneously). In a synchronous algorithm, each link proceeds to iteratively reset its rate-maximizing power strategy as if the other links were not going to change their strategies. Maintaining such an update schedule brings additional controlling overheads and weakens the applicability of distributed implementation. Therefore, we consider and propose a distributed and asynchronous algorithm.

The optimal strategy of one link, given the strategy of the other link fixed, must satisfy (8)-(11), from which we can obtain

$$p_{11}^* = \frac{\lambda_1(p_{21}^*)}{\lambda_1(p_{21}^*)+\lambda_2(p_{22}^*)}P_1 \triangleq f_{11}(\mathbf{p}_2^*), \quad (12)$$

$$p_{12}^* = \frac{\lambda_2(p_{22}^*)}{\lambda_1(p_{21}^*)+\lambda_2(p_{22}^*)}P_1 \triangleq f_{12}(\mathbf{p}_2^*), \quad (13)$$

$$p_{21}^* = \frac{\mu_1(p_{11}^*)}{\mu_1(p_{11}^*)+\mu_2(p_{12}^*)}P_2 \triangleq f_{21}(\mathbf{p}_1^*), \quad (14)$$

$$p_{22}^* = \frac{\mu_2(p_{12}^*)}{\mu_1(p_{11}^*)+\mu_2(p_{12}^*)}P_2 \triangleq f_{22}(\mathbf{p}_1^*), \quad (15)$$

where

$$\lambda_1(p_{21}^*) = p_{21}^*\alpha_{21}+\omega_{11}, \quad \lambda_2(p_{22}^*) = p_{22}^*\beta_{21}+\omega_{12},$$
$$\mu_1(p_{11}^*) = p_{11}^*\alpha_{12}+\omega_{21}, \quad \mu_2(p_{12}^*) = p_{12}^*\beta_{12}+\omega_{22}.$$

It should be noted that the above λ_1, λ_2, μ_1 and μ_2 are the aggregate interference-plus-noise terms and thus can be measured by each link locally. Next, define the vector-valued mapping function

$$\mathbf{f}_i(\mathbf{p}_{-i}) = [f_{i1}(\mathbf{p}_{-i}), f_{i2}(\mathbf{p}_{-i})]^T, \ \forall i \in \Omega.$$

Then, the iterative updating simply corresponds to performing the following iterative mapping[1]

$$\mathbf{p}_i^{(n+1)} = \mathbf{f}_i(\mathbf{p}_{-i}^{(n)}), \quad \forall i \in \Omega, \quad (16)$$

Where n is the iteration index. Accordingly, the NE is just the fixed point of the iterative mapping (16). That is

$$\mathbf{p}_i^* = \mathbf{f}_i(\mathbf{p}_{-i}^*), \quad \forall i \in \Omega. \quad (17)$$

To achieve decentralized implementations, where no signaling among different users (links) is allowed, we propose two classes of iterative algorithms: asynchronous algorithms, where the users update their strategies asynchronously according to a given schedule; and synchronous algorithms, where all the users update their strategies at the same time. Both algorithms are distributed in nature, since one user acts independently of the other to optimize its own power allocation while perceiving the other as interference. In the following, we shall elaborate on both algorithms with a formal description.

The asynchronously algorithm is actually an instance of the Gauss-Seidel scheme [16] where all the users update their own strategies asynchronously, performing the iterative mapping (17), according to Algorithm 1. The synchronous algorithm is an instance of the Jacobi scheme [16] where at each iteration, all users update their own power allocation synchronously, given the interference generated by the other users in the previous iteration, as described in Algorithm 2. For both algorithms, we have the following result:

Theorem 3: Algorithm 1 and 2 always converge to the unique NE, from an arbitrary initial point.
Proof: See Appendix B.

Algorithm 1: Asynchronous Power Control Algorithm
Set $P_i^{(0)}$ = any feasible power control, ∀i ∈ Ω;
For n =0: number of iterations

$$\mathbf{p}_i^{(n+1)} = \begin{cases} \mathbf{f}_i\left(\mathbf{p}_{-i}^{(\tau_j^i(n))}\right), & \text{if } n \in T^i, \\ \mathbf{p}_i^{(n)}, & \text{otherwise}, \end{cases} \quad \forall i \in \Omega;$$

End

Algorithm 2: Simultaneous Algorithm
Set $P_i^{(0)}$ = any feasible power control, ∀i ∈ Ω;
For n =0: number of iterations

$$\mathbf{p}_i^{(n+1)} = \mathbf{f}_i(\mathbf{p}_{-i}^{(n)}) \quad \forall i \in \Omega;$$

End

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[1]The iterative mapping (16) corresponds to a simultaneous updating rule.

5. NUMERICAL RESULTS

In this section, we present some sample numerical results.
We first illustrate the convergence properties of both algorithms through the following randomly generated channel realization, according to an exponential distribution with mean1:

$$\begin{bmatrix} |h_{11}|^2 & |h_{12}|^2 \\ |h_{21}|^2 & |h_{22}|^2 \end{bmatrix} = \begin{bmatrix} 0.7013 & 1.3206 \\ 1.5357 & 1.1982 \end{bmatrix},$$
$$\begin{bmatrix} |g_{11}|^2 & |g_{12}|^2 \\ |g_{21}|^2 & |g_{22}|^2 \end{bmatrix} = \begin{bmatrix} 1.2572 & 1.6254 \\ 1.3213 & 1.0729 \end{bmatrix}. \quad (18)$$

Without loss of generality (w.l.o.g.), we set all the noise variances to 1, and the sum power budgets $P_1 = P_2 = 1$.

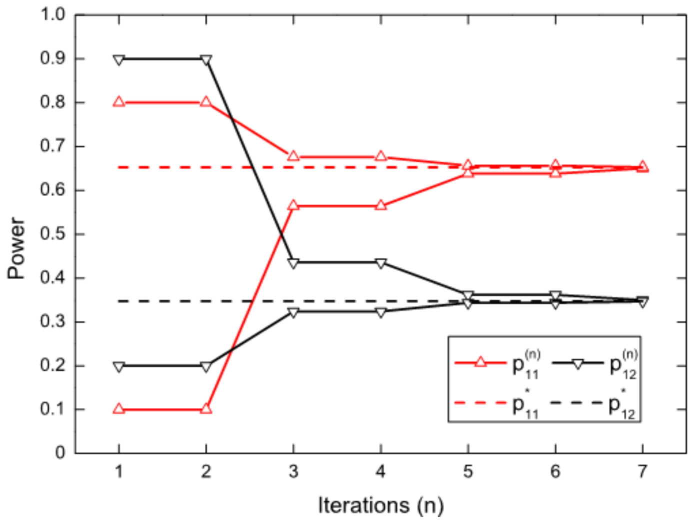

Fig.2: Convergence illustration of the sequential algorithm versus the asynchronous algorithm.

In Fig. 2 and Fig. 3 show the evolutions of the iterative power control, for both synchronous and asynchronous algorithms, from two different sets of initial points: (0 .8, 0 .2) and (0 .1, 0 9). Here we only report the power control sequence of link 1 as an example. In regard to the asynchronous algorithm, we make the assumption that link 2 proceeds with the updates twice more often than link 1

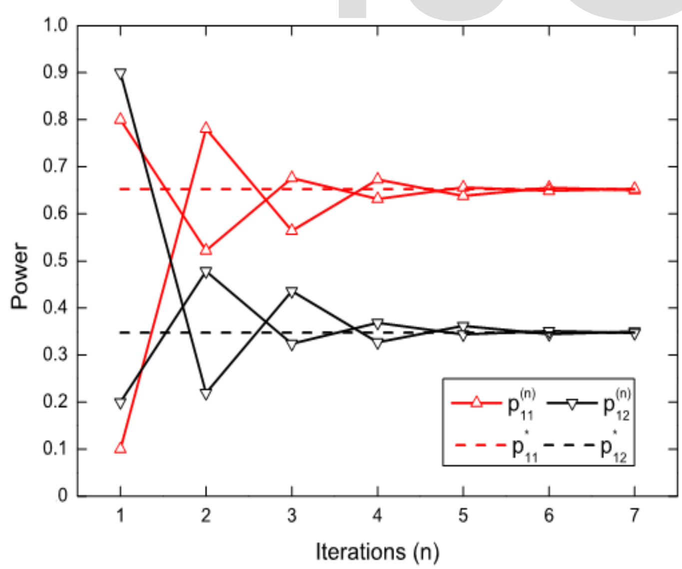

Fig.3: Convergence illustration of the simultaneous algorithm versus the asynchronous algorithm

Then, it can be observed that all the algorithms converge towards the unique NE, irrespective of the initial points. Due to space limitation, the numerical result of the case of relatively low interference settings are not shown here, yet faster convergence, as expected, is observed.

Next, we investigate the performance penalty by using the decentralized game-theoretical approach, instead of the Pareto-optimal centralized counterpart. Specifically, we compare the sum rate of two links (i.e., $I_1 + I_2$) achieved at the NE with the optimal sum rate, which can be obtained via either exhaustive search or DC (Difference of Convex functions) programming [17].

For the sake of simplicity, we consider a linear topology, where the links $S_1 \to R_1 \to D_1$ and $S_2 \to R_2 \to D_2$ forms two parallel straight lines. Besides, we set equal, the distances of $S_i \to R_i$ and $R_i \to D_i$, $i \in \Omega$, and vary the relative distance d between the two links, so that the "cross" channel gains are characterized by $|h_{ij}|^2 d^{-\zeta}$ and $|g_{ij}|^2 d^{-\zeta}$, $i = j$, $i, j \in \Omega$, respectively.

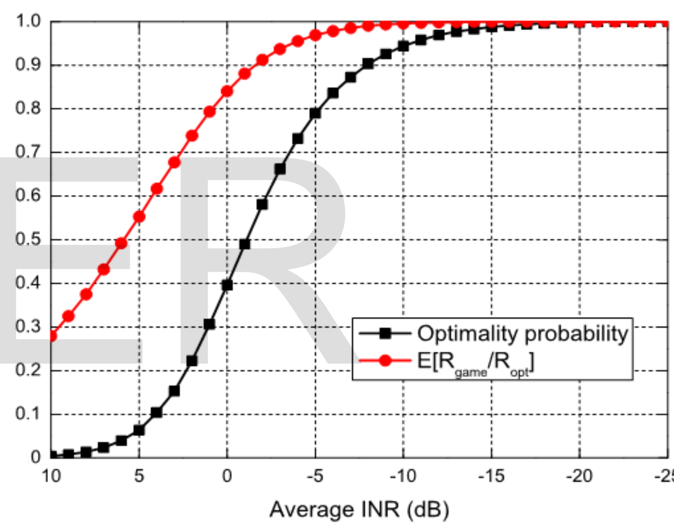

Fig.4: Optimality probability/expected rate ratio of the NE versus the average INR, with $\zeta = 3$.

Fig.4 illustrates the performance penalty of the NE in terms of the sum information rate, versus the average interference to noise ratio (INR). The squared line exhibits the optimality probability of the NE, while the circled line represents the expected rate ratio achieved by the NE over the optimal sum rate. Then, as we can see, both the optimality probability and the expected rate ratio increase, as the average INR decreases (i.e., the two links become increasingly far from each other). More importantly, as we observe, under very high interference scenarios, e.g., INR equals 10 dB, the optimality probability approaches 0, however on average, the NE still achieves 30% of the optimal sum

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rate. When the interference is moderate (e.g., the INR equals 0dB), the NE is optimal with the probability 0.4, and achieves almost 85% of the optimal sum rate on average. Thus, we can conclude that the distributed game-theoretical approach exhibits great potential in the context of interference relay channels, and qualifies as a practically appealing candidate for resource allocations.

6. CONCLUSION

In this paper, we have addressed the problem of power control in the context of interference relay channels based on a game-theoretical framework. Due to the competitive nature of the multi-user environment, we have used the framework of game theory and modeled the problem as a strategic non-cooperative game. We have shown that the proposed game always possesses a unique NE for any system profile. To allow practical implementation, we have proposed a distributed and asynchronous algorithm which has been shown to have a global convergence property. Extensive simulations have also shown that the proposed distributed method approach through extensive numerical results.

APPENDIX A: PROOF OF THEOREM 2

We prove the uniqueness of NE by showing that there exists only one feasible solution to (6)-(9). After some manipulations, p_{11}^{*} can be obtained by solving the following quadratic equation:

$$D(p_{11}^{*})^2 - (P_1 D + A + B)p_{11}^{*} + P_1 B \triangleq h(p_{11}^{*}) = 0, \tag{19}$$

Whose solution are given by

$$p_{11}^{R} = \frac{P_1}{2} + \frac{(A+B) - \sqrt{4P_1AD + (P_1D - A - B)^2}}{2D}, \tag{20}$$

$$p_{11}^{W} = \frac{P_1}{2} + \frac{(A+B) + \sqrt{4P_1AD + (P_1D - A - B)^2}}{2D}. \tag{21}$$

Where

$$\begin{aligned} D &= P_2(\beta_{12}\beta_{21} - \alpha_{12}\alpha_{21}) + (\beta_{12} - \alpha_{12})(\omega_{11} + \omega_{12}), \\ A &= P_2\beta_{21}\omega_{22} + P_1\alpha_{12}\omega_{12} + \omega_{12}(\omega_{21} + \omega_{22}), \\ B &= P_2\alpha_{21}\omega_{21} + P_1\beta_{12}\omega_{11} + \omega_{11}(\omega_{21} + \omega_{22}). \end{aligned}$$

Next, in order to show that p_{11}^{W} is always infeasible, we can simply show that

$$\left| \frac{(A+B) + \sqrt{4P_1AD + (P_1D - A - B)^2}}{2D} \right| > \frac{P_1}{2}. \tag{22}$$

Notice that A> 0 and B> 0. When D> 0, (20) is equivalent to as P_1 AD > 0, which is always true. When D<0, (20) equals to P_1 BD < 0, which is still always true. Finally when D equals 0,(17) reduces to a linear equation with the unique solution $p_{11} = \frac{B}{A+B}P_1$. The feasibility of p_{11}^{R} can be shown using the same line of proof. Thus, we prove the uniqueness of the solution p11. The same trick can be used to prove the uniqueness of other solutions. Therefore, we come to the conclusion that game G always possesses a unique NE.

APPENDIX B: PROOF OF THEOREM 3

The proof consists of two steps. Firstly, we prove the convergence of the power allocation sequence, and then show that it actually converges to the unique NE. We start with the simultaneous algorithm and define the mapping

$$\mathbf{p}_i^{(n+2)} = \mathbf{f}_i(\mathbf{p}_j^{(n+1)}) = \mathbf{f}_i(\mathbf{f}_j(\mathbf{p}_i^{(n)})) \triangleq \mathbf{T}_i(\mathbf{p}_i^{(n)}) \tag{23}$$

$\forall i \neq j,\ i, j \in \Omega.$ From (23) we observe that, the iteratively generated sequence, by taking $\{p_{11}^{(n)}\}$ as an example, is actually a combination of two subsequences $\left\{p_{11}^{(2m)} \middle| m \in \mathbb{N}\right\} = \left\{p_{11}^{(0)}, p_{11}^{(2)}, \ldots, p_{11}^{(2m)}, \ldots\right\}$, and $\left\{p_{11}^{(2m+1)} \middle| m \in \mathbb{N}\right\} = \left\{p_{11}^{(1)}, p_{11}^{(3)}, \ldots, p_{11}^{(2m+1)}, \ldots\right\}$.

A fundamental result of sequence convergence states that, a sequence is convergent if and only if all of its subsequences converge towards the same limit. Since both subsequences of $\{p_{11}^{(n)}\}$ are generated via the same mapping $\mathbf{T}_1$, we focus in the following only on $\{p_{11}^{(2m)}\}$ w.l.o.g. Denote $k = 2m$ and by substituting (16) into (23), we have

$$p_{11}^{(k+1)} = \frac{\lambda_1(f_{21}(\mathbf{p}_1^{(k)}))}{\lambda_1(f_{21}(\mathbf{p}_1^{(k)})) + \lambda_2(f_{22}(\mathbf{p}_1^{(k)}))} P_1. \tag{24}$$

Define $\delta^{(k)} = p_{11}^{(k+1)} - p_{11}^{(k)}$. After some involved manipulations $\delta^{(k)}$ can be simplified as

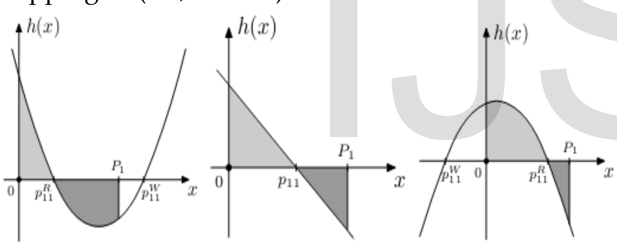
$$\delta^{(k)} = \frac{h(p_{11}^{(k)})}{\lambda_1(f_{21}(\mathbf{p}_1^{(k)})) + \lambda_2(f_{22}(\mathbf{p}_1^{(k)}))}. \tag{25}$$

Notice that the above denominator is always positive, thus the sign of $\delta^{(k)}$ just corresponds to the sign of $h(p_{11}^{(k)})$As can be seen from Fig. 5, when D> 0, h(x) is a convex function with two roots, $p_{11}^{R} \in (0, P_1)$ and $p_{11}^{W} \in (P_1, +\infty)$. When the initial point falls in$(0, p_{11}^{R})$, δ is positive, i.e. $\{p_{11}^{(k)}\}$. Is a monotonically increasing sequence. On the other hand, when the initial point lies in $(p_{11}^{R}, +\infty)$, δ is negative, i.e. $\{p_{11}^{(k)}\}$ is a monotonically decreasing sequence. When the initial point equals p_{11}^{R}, $\{p_{11}^{(k)}\}$ becomes a constant sequence. Thus, we show that the sequence $\{p_{11}^{(k)}\}$ is always monotonic and bounded $(0 < p_{11}^{(k)} < P_1, \forall k)$. Therefore, $\{p_{11}^{(k)}\}$ converges. For the cases *D* = 0 and *D* < 0, the monotonicity can be shown in the same manner and the convergence follows naturally. Therefore, we have proved that $\{p_{11}^{(k)}\}$ always converges, from an arbitrary initial point.

Next, we show that $\{p_{11}^{(k)}\}$ converges to the NE. Suppose $\lim_{k\to\infty} p_{11}^{(k)} = \xi$, we have

$$\xi = \lim_{k\to\infty} p_{11}^{(k+1)}$$
$$= \lim_{k\to\infty} \mathbf{T}_1\left(p_{11}^{(k)}\right) \overset{(a)}{=} \mathbf{T}_1\left(\lim_{k\to\infty} p_{11}^{(k)}\right) = \mathbf{T}_1(\xi), \quad (26)$$

Where (*a*) is justified by the continuity of the mapping function $\mathbf{T}_1$. Therefore, as long as $\{p_{11}^{(k)}\}$ converges, it is guaranteed to converge to the fixed points of the mapping $\mathbf{T}_1$ (i.e., the NE).



(a) when $D > 0$ (b) when $D = 0$ (c) when $D < 0$

Fig. 5: $h(x)$ versus x.

The same trick can be used to prove the convergence of the other subsequence of $\{p_{11}^{(n)}\}$. Therefore, $\{p_{11}^{(n)}\}$ converges to the unique NE. Convergence of other power sequences can be similarly proved. This completes the convergence proof of the simultaneous algorithm.

Finally, we note that the convergence also holds for the sequential algorithm. Because the power sequence is composed of two identical sequences, each of which can be proved to converge to the unique NE by the same approach. Hence, we have proved that both algorithms always converge to the unique NE of game G.

REFERENCES

[1] G. J. Foschini and Z. Miljanic, "A simple distributed autonomous power control algorithm and its convergence," *IEEE Trans. Veh. Technol.*, vol. 42, no. 4, pp. 641-646, Nov. 1993.

[2] R. D. Yates, "A framework for uplink power control in cellular radio systems," *IEEE J. Select. Areas Commun.*, vol. 13, no. 7, pp. 1341-1347, Sept. 1995.

[3] A. Gjendemsjø, D. Gesbert, G. E. Øien, and S. G. Kiani, "Binary power control for sum rate maximization over multiple interfering links," *IEEE Trans. Wireless Commun.*, vol. 7, no. 8, pp. 3164-3173, Aug. 2008.

[4] M. Chiang, C. W. Tan, D. P. Palomar, D. O. Neill, and D. Julian, "Power control by geometric programming," *IEEE Trans. Wireless Commun.*, vol. 6, no. 7, pp. 2640-2651, July 2007.

[5] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge University Press, 2004.

[6] Z.-Q. Luo and S. Zhang, "Dynamic spectrum management: Complexity and duality," *IEEE Trans. Signal Processing*, vol. 2, no. 1, pp. 57-73, Feb. 2008.

[7] M. Xiao, N. B. Shroff, and E. K. P. Chong, "Utility-based power-control scheme in cellular wireless systems," *IEEE/ACM Trans. Networking*, vol. 11, no. 2, pp. 210-221, Apr. 2003.

[8] K. K. Leung, C. W. Sung, W. S. Wong, and T. M. Lok, "Convergence theorem for a general class of power control algorithms," *IEEE Trans. Commun.*, vol. 52, no. 9, pp. 1566-1574, Sept. 2004.

[9] R. K. Mallik, R. A. Scholtz, and G. P. Papavassilopoulos, "Analysis of an on-off jamming situation as a dynamic game," *IEEE Trans. Commun.*, vol. 48, no. 8, pp. 1360-1373, Aug. 2000.

[10] W. Yu, G. Ginis, and J. M. Cioffi, "Distributed multiuser power control for digital subscriber lines," *IEEE J. Select. Areas Commun.*, vol. 20, no. 5, pp. 1105-1115, June 2002.

[11] Z.-Q. Luo and J.-S. Pang, "Analysis of iterative waterfilling algorithm for multiuser power control in digital subscriber lines," *Eur. J. Appl. Signal Processing*, no. 1, pp. 1-10, Apr. 2006.

[12] G. Scutari, D. P. Palomar, and S. Barbarossa, "Optimal linear precoding strategies for wideband non-cooperative systems based on game theorypart I: Nash equilibria," *IEEE Trans. Signal Processing*, vol. 56, no. 3, pp. 1230-1249, Mar. 2008.

[13] R. Etkin, A. Parekh, and D. Tse, "Spectrum sharing in unlicensed bands," *IEEE J. Select. Areas Commun.*, vol. 25, no. 3, pp. 517-528, Apr. 2007.

[14] E. C. V. D. Meulen, "Three-terminal communication channels," *Adv. In Appl. Probab.*, vol. 3, pp. 120-154, 1971.

[15] T. Cover and A. E. Gamal, "Capacity theorems for the relay channel," *IEEE Trans. Inform. Theory*, vol. 25, no. 5, pp. 572-584, Sept. 1979.

[16] A. Sendonaris, E. Erkip, and B. Aazhang, "User cooperation diversity– part I: system description," *IEEE Trans. Commun.*, vol. 51, no. 11, pp. 1927-1938, Nov. 2003.

[17] J. N. Laneman, D. N. C. Tse, and G. W. Wornell, "Cooperative diversity in wireless networks: efficient protocols and outage behavior," *IEEE Trans. Infom. Theory*, vol. 50, no. 12, pp. 3062-3080, Dec. 2004.

[18] O. Simeone, O. Somekh, Y. Bar-Ness, H. V. Poor, and S. Shamai, "Capacity of linear two-hop mesh networks with rate splitting, decodeand- forward relaying and cooperation," in

Proc. 45th Allerton Conf. Commun., Control Computing, Sept. 2007.

[19] V. I. Morgenshtern and H. Bölcskei, "Crystallization in large wireless networks," *IEEE Trans. Inform. Theory*, vol. 53, no. 10, pp. 3319-3349, Oct. 2007.

[20] J. W. Huang, R. A. Berry, and M. L. Honig, "Distributed interference compensation for wireless networks," *IEEE J. Select. Areas Commun.*, vol. 24, no. 5, pp. 1074-1084, May 2006.

[21] M. O. Hasna and M.-S. Alouini, "Optimal power allocation for relayed transmissions over Rayleigh-fading channels," *IEEE Trans. Wireless Commun.*, vol. 3, no. 6, pp. 1999-2004, Nov. 2004.

[22] Y. Yao, X. Cai, and G. B. Giannakis, "On energy efficiency and optimum resource allocation of relay transmissions in the low-power regime," *IEEE Trans. Wireless Commun.*, vol. 4, no. 9, pp. 2917-2927, Nov. 2005.

[23] M. Osborne and A. Rubinstein, *A Course in Game Theory*. Cambridge, MA: MIT Press, 1994.

[24] D. P. Bertsekas and J. N. Tsitsiklis, *Parallel and Distributed Computation: Numerical Methods*. Athena Scientific, 1989.

[25] G. Scutari, D. P. Palomar, and S. Barbarossa, "Asynchronous iterative waterfilling for Gaussian frequency-selective interference channels," *IEEE Trans. Inform. Theory*, vol. 54, no. 7, pp. 2868-2878, July 2008.

[26] D. P. Bertsekas, *Nonlinear Programming*. Athena Scientific, 1995.

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